Researching the 3-D Complex Plane

October 2018



http://iMathTools.com

Introduction

I am Dave Fashenpour; a retired USAF Officer, a retired Boeing Embedded Software Engineer on the International Space Station, and a four-year veteran Math Teacher from Houston, Texas. I have a Bachelor of Science degree in Mathematics from Colorado State University dated 1970 and a Master of Science degree in Computer Science from the University of Southern Mississippi dated 1977. My current business is the Instructional Math Tools, an LLC from Texas relocated to Florida.

Part1

We will examine both the 2-D Mandelbrot Baby and get the first glimpse of the 3-D Log. We also will reveal a technique to add a second Imaginary Plane to the standard Complex Number Plane; having just one Imaginary Plane.

In the late 1970's, Benoit Mandelbrot devised a **set** of <u>complex numbers</u> that he called 'C' and he was fascinated when that set of numbers caused a specific algebraic function **not** to diverge. Divergence here, meant the results were erratic and sometimes explosive – escaping to infinity. His set of numbers, 'C', tamed the loop-back value of the iterative equation and focused the cyclic results into a stable condition and many times even disintegrated into a very small quantity – even zero.



Figure 1- The Traditional Mandelbrot Baby

Figure-1, shows the <u>tamed</u> results as black points (pixels), with the <u>erratic</u> results as purple pixels; but the fringe-area in-between the tamed & erratic results (i.e. the reddish and yellowish pixels) are the most interesting and the only reason we are talking about fractals in the first place. Fractals are naturally occurring geometric designs that are breathtakingly beautiful; and just happen to live in the border-lands between the black pixel island (Mandelbrot Set Members) and the purple pixel sea (Set Rejects).

The significant factor is <u>how many times</u> did the iterative loop had to loop-back on itself to determine if the result was erratic or tamed. Erratic loop-back numbers jump out of the iterative loop earlier than the others jump out (perhaps 40 loops or less). The more stable the result, the longer it takes to jump out; in fact, true members of the Mandelbrot Set (black pixels) never jump out – but loop forever. Fractals live between 40 and forever.

To avoid waiting 'forever', one can place a limit on the loop-back maximum to something reasonable, like 10,000 or 100,000 loops. As the iterative loop-count approaches the loop-back max, the selected value of 'C' looks more and more like a true member of Mandelbrot's Set of complex numbers. We then color the original point 'C' as a black pixel, if and only if, the loop-count hits the max value, signifying the unmistakable membership into Mandelbrot's "club" of complex numbers.

What about the pixels that are **not** black or purple? Where shall we draw the lines? An arbitrary lower-limit range is 1 through 40 loop-backs to qualify as a purple pixel. Then an upper-limit range of let's say 10,000 to qualify as a (Set Member) black pixel; with 10,000 being totally arbitrary. The loop-back counts between 40 and 10,000 are fractals (neither black nor purple). The range limits of 40 and 10,000 -- even the chosen <u>pixel</u> <u>colors</u> - are **completely arbitrary**. Your choice of 'green' instead of 'purple' and your lower limit of '50' instead of '40' would work just fine. Your baby will have observable differences, but the basics never change. I have coined a new term - MandelFash - and have established a new technique to deal with the Mandelbrot Set. While I remain well within Mandelbrot's complex world, other folks depart from his magic and use Quaternions and other Spherical Coordinate Systems to manufacture what <u>they</u> believe to be a 3-D Mandelbrot "Baby"; but they are not even in the same 'complex' universe. These folks have removed themselves from Mandelbrot's complex coordinate systems and used non-complex, foreign contraptions to video pretty pictures and vivid exploratory trips into the innards of a 'baby' – but these are not 3-D Mandelbrot Babies!

How to add an Additional Imaginary Plane

Mandelbrot defined his complex number as C = a + bi, which is a point on a two-dimensional complex, flat plane. If you try to add a conventional axis, such as the Z axis to acquire a 3-D view – you will find yourself with dead-end algebraic components, not complex variables. The Mandelbrot equation requires you to multiply the chosen points together, among other things. This squaring methodology renders conventional 3-D approaches useless -- which is one of the reasons that no one has created a true 3-D Mandelbrot Baby, until now. Another reason why most people have not succeeded, is because the wisdom of the internet states: "there is no 3-dimensional analogue of the 2-dimensional space of complex numbers".

<u>Mandelbrot</u>



Figure 2- A minor addition to C

What is so special about what I did? Leaving the iterative equation exactly as Mandelbrot designed it, I successfully added another dimension to Mandelbrot's <u>definition of C</u>; see Figure-2. This breakthrough approach added a complex plane that I called "i-prime" to the original "i" plane. Mandelbrot's original 1979 equation, along with his definition of the complex number C is illustrated in Figure-2. His definition of C needed a "little boost" to enter a 3-D complex world. That's it! That is all I did; except for also redefining the arithmetic rules of multiplying "i-prime" and "i". We know that 'i' times 'i' is negative one – but what is 'i-prime' times 'i-prime' (that detail became an integral part of my newly devised rules for multiplying 'i-prime').

So why hasn't someone done this before now? This quote, taken from the Mandelbulb website, says it all:

"The Mandelbulb is a three-dimensional fractal, constructed by Daniel White and Paul Nylander using spherical coordinates in 2009. A canonical 3-dimensional Mandelbrot set does not exist, since there is no 3-dimensional analogue of the 2-dimensional space of complex numbers. But it is possible to construct Mandelbrot sets in 4 dimensions using quaternions."

The requirement to devise new multiplication rules, plus statements like the above Mandelbulb quote, were stumbling blocks to creating a 3-D Mandelbrot Baby. The iterative algebraic equation that was Mandelbrot's masterpiece, requires you to SQUARE whatever complex number you pick. So, how do you square: [2.4567 + 3.9504i + (-7.1236i')]? At the heart of the definition of the arithmetic involving Complex Numbers, is that 'i' times 'i' is -1. However, think about what is the meaning of 'iprime' times 'i'? That is why I had to invent new math rules. See Figure-3. MandelFash Rules for i-Prime

```
Rule #1: i * i = -1

Rule #2: i' * i' = -1

Rule #3: i' * i = -1

Rule #4: i * i' = -1

Rule #5: -1 * i = -i

Rule #6: -1 * i' = -i'

Rule #7: (i * i) * i = -1 * i = -i

Rule #8: (i' * i') * i' = -1 * i' = -i'

Rule #9: i * (i * i) = i * (-1) = -i

Rule #10: i' * (i' * i') = i' * (-1) = -i'
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Figure 3- New Math Rules

I proceeded to square those two complex numbers (both with an 'i' and an 'i-prime') BY HAND, using the newly invented Rules for 'i-Prime'. Ouch! It was painful – even with the new rules. But, as I did it over and over again; I realized that there was a consistent pattern. Certain quantities were always added or multiplied together, and the result always was a series of REAL numbers that needed to be added and there was typically one number dedicated to 'i' and another dedicated to 'i-prime'. So, all I had to do was to add and then rearrange them into a final answer.

Once the pattern was set, I had to write a software program that would recognized my new math rules and the rest was easy. After testing the finished product, I began seeing loop-counts that suggested there were actual SET MEMBERS for the MandelFash Set of Complex Numbers. This led me to believe I was on the right-track. Now I knew that my MandelFash Rules for i-Prime were valid and that those rules had led me to the discovery of a previously unknow entity. See Figure-4 to see the pattern used to square two triplets, both of which are composed of 3-D Complex Variables.



Figure 4- Squaring with New Math rules

Squaring a <u>triplet</u> was NOT in the prescribed calculations of Mandelbrot's classic formula. I took great care only to customize an absolute minimum of the pure Mandelbrot process; inverting 4 arithmetic signs, making a plus into a minus and making a minus into a plus sign. Those changes were to accommodate the 'i-prime' variables. Figure-5 illustrates the actual Visual Basic code that implements this new recursive equation. It worked, but it was no longer pure Mandelbrot, so I christened it the MandelFash Recursive Formula, © 2018.

Customization of Mandelbrot's Recursive Formula

```
Savx = x : Savy = y : Savz = z
For i = 1 To 20
  z1 = (z^{2})
                          <= Real Component – SIGN CHANGE
  z^{2} = (z * y)
                          <= Real Component – SIGN CHANGE
  z3 = (z * x)
                          <= Imaginary Component #2 (I')
  y1 = (y * z)
                          <= Real Component – SIGN CHANGE
  v^2 = (v^2)
                          <= Real Component – SIGN CHANGE
  y3 = (y * x)
                          <= Imaginary Component #1 (I)
  x1 = (x * z)
                          <= Imaginary Component #2 (I')
  x^{2} = (x * y)
                          <= Imaginary Component #1 (I)
  x3 = (x^{2})
                          <= Real Component
  w1 = x3 - y2 - y1 - z2 - z1
                             <= Real Number (with sign changes)
  w^2 = x^2 + v^3
                              <= Existing Imaginary (I)
  w3 = x1 + z3
                              <= Existing Imaginary (I')
  x = w1 + Savx
                          <= Real Number Add Back
  y = w2 + Savy
                          <= Existing Imaginary (I) Add Back
  z = w3 + Savz
                          <= Existing Imaginary (I') Add Back
Next
```

Figure 5- The MandelFash Recursive Formula

The strange graphic image that was the result of executing this new recursive formula, was a "Rugged-Looking Log" of many colors, see Figure-6. It was extending as much as four units from zero in both directions; projecting upward toward the negative 'i-prime' axis and downward toward the positive 'i-prime' axis. The Log's projection 'leaned' at an angle of 45-degrees to both the X-Y plane and the X-Z plane. I did not know what the 'Log' was or what to do with it.

I was a little disappointed not seeing an image of a baby and wondered if this result was worth the amount of effort that I had exerted. I decided to take a graphic slice off both ends of the Log; at ½ units on either side of zero, so that I could see what was on the inside. When I turned the Log around, I saw things that got me excited.



Figure 6- My Rugged-Looking Log

The log-cutting software had revealed a colorful cavity inside the Log and a yellow 'wormhole' that joined the upper & lower portions of the Log; see Figure-7. The side-profile of the cut-off Log had the approximate shape of a Mandelbrot Baby and if you looked closely on the far-side, you could make-out a miniature baby living on what looked like a proboscis. I believed that I had discovered my MandelFash Baby.

This 'Baby' had what appeared to be fractals covering its surface and upon closer examination, I discovered a wonderland of beautiful and diverse fractal structures covering the entire Log. At this point, one could imagine new, never-before-seen fractals being revealed, with each new slice of the Log. Cutting very thin slices of the Log could reveal an infinite amount and an astonishing variety of fractal silhouettes. Most Mandelbrot enthusiasts are not mathematicians and are just seeking pretty, self-replicating fractal designs. To these casual observers, the mathematics behind those shapes are not that important – especially since anyone can download a fractal generator tool and create their own videos, with minimal mathematic knowledge. While I too, appreciate the vivid colors and fantastic tapestry that joins together the fractal veins of my MandelFash Baby, I also carry the burden of making sense out of a graphic experience that was born out of curiosity and stubbornness.



Figure 7- Side of the Log at 45-Degrees

This graphic figure was generated by selecting complex variables in a 3-D Complex Plane; which is the mathematical significance of this discovery. Yes, the fractals are beautiful and interesting – but the math used behind this newly discovered image incorporates an original, copyrighted, technique that fuels the MandelFash Recursive Formula to locate set members and the border fractals that surround those set members.

Part 2

We will explore the slicing of the MandelFash Fractal Log and reveal the 2-D Mandelbrot Baby, hidden within the Log.

How do you slice or cut-into a geometric structure that appears to be a 'rugged log', leaning at a 45-degree angle? The Log slopes down and penetrates right-through the 2-D horizontal plane – at the exact location of the original Mandelbrot Baby! Evidently the extension of a second complex plane to the original Mandelbrot 2-D quadrants, forced the baby to be projected up at a 45-degree angle and down, below the baby at a 45degree angle. Designing a mathematical 'cutter' involved a 2-D plane moving through the 3-D coordinate space, while recording all points in common and sending those coordinate points to the recursive formula for determination about their set membership.



Figure 8 - Side View of Log

Turning the truncated Log 45-Degrees up toward the positive Z axis, exposed a silhouette that I had been looking for. The Log's side-view in Figure-8, was in the shape of a Baby, with a yellow 'wormhole' and it even revealed a proboscis. Now things were starting to get interesting. I knew that I had to slice the Log yet again, but this time producing a thinsliced 2-D Baby for examination. I programmed the flat 2-D plane that intersected with the Log vertically. The result was a thin slice consisting of all the coordinate points that belonged to both the 3-D geometric Log image and the 2-D vertical 'cutter' plane. These shared-points in space were given to the MandelFash Recursive Formula and in-return, the color assignments for each point were received and plotted. Black represented a Member of MandelFash Set, while (in this case) the background color assignment was also Black. The area between the Set Members and the background color, were the reds, yellows, and the purples of the surface fractals. See Figure-9.



Figure 9- The Zero Slice

This slice was achieved with only 20 iterations of the recursive loop, while the regular maximum limit is set to something above 1,000. People who assign a limit on the iterative loop of 100 or less are not serious – but what did I know? I had found a Log and wanted to quickly find out what was inside this mysterious MandelFash Fractal Log.

I was very proud of this 'zero-slice' baby (aka MandelFash Baby). With a recursion limit of only 20, the baby was beautiful, and had all the basics of the more sophisticated babies. It had fractals, it had a cute cardioid-shaped body, and most surprising – there was a tiny baby living on its proboscis.

As I learned more about 'the loop', I increased the loop limit to 500 (even though today I think 10,000 is a low number). This sophisticated processing, along with a greater sensitivity to color variations of the red & black baby – turned into a much more elegant image. Assigning gold and maroon color codes to early breakouts of the MandelFash Recursive Formula, may have not been the traditional Mandelbrot color scheme, but I thought my little Baby was fantastic. See Figure-10.



Figure 10- Zero Slice with Higher Loop Limits

This center-slice of my MandelFash Fractal Log not only looked amazingly like a Mandelbrot Baby – it turned-out to be even more beautiful than the original 1980 version, because this one had a golden hallo, giving a seemingly 3-D foundation for the fractals to live and prosper on the surface of the Log. The fractals that covered the fractal log, were in fact, identical to the fractal types surrounding the Mandelbrot Baby – with the only difference being the closer the fractals were to the end of the Log, the more degraded they became; until finally dissolving into black pixels (Set Members) at the extreme ends of the Log.

It was not a Mandelbrot Baby at all – it was one of the thousands of MandelFash Baby slices that makeup the MandelFash Fractal Log, shown in Figure-11. In fact, when you stack-up the many slices of the Log about one-half unit apart, you get an idea of what a solid Log might look like.



Figure 11 – MandelFash Fractal Log, ©2018

The image of the MandelFash Fractal Log in Figure-12, along with the mathematics which produced that image, are all included in a 2018 Copyright, by Instructional Math Tools, Melbourne, Florida. The MandelFash Fractal Log consists of an infinite number of MandelFash Babies and is in a **three-dimensional complex number plane** which surrounds and engulfs the famous two-dimensional Mandelbrot Baby.

As you can see, there are slices of babies from top to bottom and combine to form the Log. Notice also, in the center of the Log is a black core, made-up of MandelFash Set Members and are represented by black pixels. If you look closely, you can observe the Vertical MandelFash Baby resting comfortably in the middle of the Log – sporting a very handsome proboscis; intersecting though the central axis of that baby is the Horizontal MandelFash Baby – also known as the Mandelbrot Baby.



Figure 12 - Stacking the Babies

Each of the slices have a fractal 'skin' around the edges and forming a border area between set members and non-set members. Sandwiched in the middle is a black core (set-member pixels), which runs through the middle of the log – even extending past the fractals and forming black 'wings' on each side of the Log.



Figure 13 - Fractal Feathers Top & Bottom

In Figure-13, you can see the fractal 'feathers' on the top of and beneath each black wing-tip. The fact that this newly discovered entity resembles a space-ship from the future may add some mystery to this image; especially when you consider many Mandelbrot enthusiasts claim that the Mandelbrot's baby is in fact, the "thumb-print of God".

What started-out two years ago, trying to discover the true appearance of my "Rugged-Looking Log", finds us examining this very interesting shape. I know that my mathematics is revolutionary, mainly because if someone else had discovered this Log; we all would have heard about it before now. I am glad that I pressed-on, considering everyone told me that there was no 3-D Complex Number Plane.

The MandelFash Fractal Log is not only curious, but it is a unique entity that has never been observed before and needs to be the subject of intense research by my fellow mathematicians, to validate and to verify that which I have claimed, is in-fact the truth. Recent 360-degrees radial scans surrounding the lateral axis of the original Mandelbrot Baby, proves that the MandelFash Fractal Log <u>is</u> the illusive three-dimensional Mandelbrot Baby. Sweeping a graphic straight-line, rotating it through the axis that runs through Mandelbrot Baby, in all 4 quadrants, revealed two 'normal' quadrants and two 'radical' quadrants.



Figure 14 - Quadrant Sweeps

The results were surprising in that the sweep on the left-side of Figure-14 was totally expected (only up to 1.0 on the Z-Scale); while the sweep on the right-side of Figure-14 was way too big (up to 2.8 on the Z-Scale). The 3-D Baby was well-behaved in Quadrants #2 & #4. But in Quadrants #1 & #3, the Set Pixels (black) were extending their wings and becoming a Log at a 45-degree angle up and down the Z-Scale. This unexpected phenomenon confirmed that what was being assembled was the 3-D Mandelbrot, the MandelFash Fractal Log.

The radial scan produced over 2,500 'CSV' files (Excel comma-delimited files) and inside each of those files there was 100,000-pixel locations defined (with an X, Y, & Z) and a color code for each of those pixels. That massive amount of data had an unintended consequence; it was too big for my computer and only produced a partial final product, shown in Figure-15.

I thought of Benoit Mandelbrot in 1980, getting full control of an IBM 370 Mainframe for the first time. He stretched the limits of that mainframe, just like I am stretching the limits of my environment. Mandelbrot produced a picture of a fractal baby that changed the world of mathematics. Would my meager efforts produce something of any value? The jury is still out on that question. The Figure below appeared right before my computer quit.



Figure 15 - Partial View of a 3-D Baby

I have frozen my computers multiple times, tried to divide up the massive data files, written intelligent pre-processor programs to replace extra data points, and even asked the maker of the Graphics Computer 3-D software to modify his code – all to bring you a picture. Figure-16 is a start, with about half of the data files processed, I got a glimpse of my target image. My work was not done – not until I could show you an up-close detailed view of the famous 3-D Mandelbrot, which by the way, is looking more and more like a MandelFash Fractal Log than a Mandelbrot Baby.



Figure 16 - One-half of the Data Files Loaded

Part 3

We will examine the evolution of the MandelFash Fractal Log, starting with the "Blue Goose", the "Naked Log", and the "Sliced Log".

I decided to change my tactics, instead of concentrating on the threedimensional space surrounding the Mandelbrot Baby; I decided to attack the 'elephant-in-the-room', the three-dimensional space five units out from zero, in all directions. The purpose of that attack was to obtain a 'snap-shot' of the entire MandelFash Fractal Log.



Figure 17 - The "Blue Goose"

The 'Blue Goose' was massive, because the graphic data generated by searching for objects within 5 units out from zero; required over 24 Giga Bytes (GB) of RAM memory, just to display the above image.



Figure 18 - Blue-Skinned Baby

My computer did not have enough RAM – my Lap Top had 8 GB and my ASUS gaming machine had 16 GB. The graphics program died every time the 'Blue Goose' image started to form on the screen. I bought another computer, a special 64 GB RAM OptiPlex Mini-Tower from Dell. I held my breath when the final image began to form on the screen. I loved it at first, but soon grew to hate the blue "skin", which was consuming the entire image. It looked okay on a baby, see Figure-18, but it became painfully obvious that the blue had to go – I needed to see the surface fractals that were living on the 'Blue Goose'. It was my own fault, because I had specified a blue color trio of assignments to all EARLY-BREAKOUTS from the recursive loop. This technique was okay for individual babies, but not for the Log. I had to skin this 'Blue Goose'. I re-ran the data collection program that had created the 'Blue Goose'; but this time I assigned NO COLOR to all the early breakouts from the recursive loop. I produced the 'Naked Log' in Figure-19.



Figure 19 - The Naked MandelFash Fractal Log

The Naked MandelFash Fractal Log is a solid 3D object that surrounds the core graphic, the Mandelbrot Baby. The log is made-up of an infinite number of stacked MandelFash Babies with fractals all around, until they mutate into BLACK PIXELS at the end of the 'wings'.

For over 38 years people have tried to produce a 3D Mandelbrot, but all of them have FAILED. They hired artists and drew elaborate conceptual designs of what they THOUGHT was the 3D Mandelbrot. Programmers modified the equations that Mandelbrot had developed and bragged their new graphic was the 3-D image! But they were <u>all wrong</u>. WHY? They were all wrong because they had divorced themselves from the Complex Number Plane. The Naked MandelFash Fractal Log consists exclusively of only COMPLEX VARIABLES, as does the Naked Baby in Figure-20.



Figure 20 - Naked Baby, a Vertical Slice

To examined this newly discovered Naked Log, I had to slice it. The Sliced MandelFash Fractal Log had a density of ten babies per unit, with at least 44 sliced babies on each side of zero; see Figure-21. The 'Sliced Log' consists of an infinite number of MandelFash Babies, where fractals surround each slice; that is until the slicing approached the end of the wings; where they mutate into black pixels. At the origin, where the Mandelbrot Baby lives, the center-baby in the 'Sliced Log' proudly displays a fine mini-MandelFash Baby on its proboscis.



Figure 21 - The Sliced MandelFash Fractal Log

There are three (3) significant slices from the MandelFash Fractal Log. There is the famous DIAGONAL slice that cuts the log in half, there is the VERTICAL slice which slices this 45-degree leaning log at 45-degrees, and finally there is the HORIZONTAL slice which again attacks this 45degree leaning log at a 45-degree cut (90 degrees from the vertical).



Figure 22 - Diagonal Cross-Section of the Log

This 'Diagonal Slice' of the MandelFash Fractal Log; has different measurements when compared to the Horizontal and to the Vertical slices. Figure-22 shows this classic profile snapshot of the cross-section of the MandelFash Fractal Log. Examining the stacked babies in Figure-22, you can clearly see the mutating babies, whose fractals decay into black Set Members. One can observe the Nose mutating and finally vanishing. The Neck first turning color and then separating totally from the body. The Top and Bottom Fins also degrade and finally vanish into black pixels. It is like the fractals give-out, but the baby's body & head continue-on and turn-into wings.



Figure 23 - Diagonal Slice

Figure-23 shows a 'Diagonal Baby', with a proboscis and a mini-baby, and is the official profile 'shape-of-the-Log'. This shape is significant because it creates the Horizontal and Vertical babies as it collides with the X-Z and X-Y coordinate planes.

The 45-degree collision of the 'Diagonal Baby' into the X-Y Plane results in a 'Horizontal Baby' and when it collides with the X-Z Plane at 45degrees, the resultant image is a 'Vertical Baby'. That begins to explain the significance of the MandelFash Fractal Log. More detail later.



Figure 24 - Horizontal Slice

Figure-24 is the image of the Mandelbrot Baby, but it is really a 2-D Horizontal slice of the MandelFash Fractal Log and lives at Z=0.



Figure 25 - Vertical Slice

Figure-25 is the image of a 2-D Vertical slice of the MandelFash Fractal Log and lives at Y=0. It is an exact copy of the Horizontal baby which lives on the X-Y Plane; but this Vertical baby lives on the X-Z Plane. The vertical image is the sibling of the Mandelbrot Baby – the baby brother of the horizontal image.

The 'stars' of the show are the 'Horizontal Slice', the 'Vertical Slice', and the 'Diagonal Slice' of the MandelFash Fractal Log. While the first two are statistically equivalent; the 'Diagonal Slice' is another story altogether.

The 'Diagonal Slice' is the exact shape of the center of the Log (if you were to cut the Log in half). This shape deteriorates from the center of the Log and becomes decayed and distorted finally into the black wings.

But as I said previously, this shape is truly amazing because of the impact it has on the X-Y and X-Z Planes as it penetrates those planes. This elongated, stunted baby is situated at a 45-degree angle from both the Yaxis and the Z-axis, slips through those respective planes in such a way as to leave the famous Mandelbrot Baby's shape in its wake. What I am saying is that this ugly Diagonal, produces two beautiful babies, one on the X-Z Plane and one on the X-Y Plane.

As a final note, one must address the mutation of the baby's shape as it migrates from the Log center to the end of Log's wings. The Log slices depicted in Figure-26 are divided into 14 segments with 7 segments on each side. Notice the loss of the proboscis, from frame #0 to frame #1.



Figure 26 - Mutation Progress

Observe the Nose from frame #0 to frame #3. Observe the Head from frame #0 to frame #7 (detached and shrunk). Observe the Body from frame #0 to frame #7 (leaving just a lump of coal). But please do not forget about the fractals.

Those beautiful fractals which undergo severe degradation from frame #0 to destruction in frame #7, not only get destroyed, but as the mutation progresses from center to the end of the wings, they undergo modifications. They become different with changes in coloring and evolving new characteristics.

We do have tools that will allow you to explore the Log's fractal degradation for yourselves – the tools are available on the Instructional Math Tools website: iMathTools.com.

If you do pursue our fractal explorers, you will see what I call "Fractal Hell", an environment that documents the stress and the mutation that the fractals experience. You will see what appears to be fire surrounding various min-babies and you will see the reduction in fractal life-lines. Life-lines stem from a central Mini-baby and thrust out into the adjacent fractal space, seemingly to create new babies, almost looking like 'nutrients' flowing from parent to child. But in the stressful environment of wing mutation, the nutrients seem much more limited and constrained – resulting in fewer mini-babies.

Part 4

We will see why the MandelFash Fractal Log is significant; by demonstrating how it interacts with the 2-D Mandelbrot environment.

The Log crashes through the X-Y and the X-Z Plane in a very exciting manner. The MandelFash Fractal Log is shown as it exists at a 45-degree angle to both the X-Y and to the X-Z Plane. The center-point in the middle of the Log is the intersection of two lines: 1) Y=0, which is the center-line of the Mandelbrot Baby – from nose to the butt of a Horizontal Baby; and 2) Z=0, which is the center-line of the MandelFash Baby – from nose to the butt of a Vertical Baby. See Figure-27.



Figure 27 - MandelFash Fractal Log & the X-Y Plane

Figure-28 shows the two babies intersecting (in red), where both the 'Horizontal Baby' and the 'Vertical Baby' meet at their center-lines. The red baby that is aligned left-to-right, is in fact the original Mandelbrot Baby. One can readily see the interconnect between Mandelbrot's 2-D world the MandelFash 3-D world.



Figure 28 - Crossing the Mandelbrot Baseline

The above figure shows the central babies enlarged and in red to indicate their relative position to the MandelFash Fractal Log. In Figure-29 the babies are shown in their actual size and color. It is very easy to see the delicate alignment and interconnect between the Log and the Babies; whereas the extreme tips of the babies fit perfectly into the edges of the Diagonal Babies. The Log penetration of both the X-Y and X-Z Planes, is the driver that carves-out two central babies from the stacked Diagonal babies within the MandelFash Fractal Log.



Figure 29 – A Perfect Fit

One cannot examine the famous Mandelbrot Baby (horizontal slice), without extending that examination to the Mandelbrot Sibling (vertical slice). This is because they are indelibly linked together by two dimensions: where the 2-D babies join to birth a third dimension. Conversely, one cannot study the pair of central babies, without considering the MandelFash Fractal Log's creative effect of producing the babies.

To put it another way, if you ignore the MandelFash Fractal Log, you have seriously missed a dominate feature of Mandelbrot's world. It was not Mandelbrot's fault for stopping short at his 2-D creation, because he not only had limited computer resources (IBM 370), but he also had just turned-on a 'fire-hydrant' supplying more information than could possibly be consumed by the math community.

Mandelbrot was satisfied that his discovery had advanced the field of fractals greatly and he had also opened-up the Coastline Paradox; a counterintuitive examination of England's Coastline; where the world of unmeasurable distances, due to infinitesimal measuring sticks, related very nicely to his world of fractals. He was very content with his 1980 conclusions, but I am also content that my 2018 conclusions have expanded his great work. The only travesty is for someone to ignore this new information and live in the past. Science and Mathematics are not as static as some scholars would have you believe. Is everybody so content with existing textbooks that new ideas are rejected? Or are we still thinking out of the box and exploring brave new worlds?

Please join my quest and prove me wrong, with an explanation. Better yet, prove me right and help me write a new chapter in the math books of the future. This new technique may apply to different fields of study and this approach may even open some doors that have remained closed for many years. When Complex Numbers are required – what advances could be accomplished by applying the MandelFash approach to achieving 3-D graphics of those problem-solutions that seem to be just out of reach. That was my first conclusion two years ago – this solution is just out of reach; but today I have a working hypothesis, one that actually creates beautiful 3-D graphics and hopefully has expanded the magic conjured-up over 38 years ago.

My final Figure-30 displays the product of our most recent Fractal Explorer Tool, we took the Naked fractal environment and added a blue background. Welcome aboard the MandelFash experience.



Figure 30 - Blue Background